



Distributed Detection of a Non-cooperative Target with Multiplicative Fading

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Abstract. We tackle distributed detection of a non-cooperative target with a Wireless Sensor Network (WSN) made of tiny and inexpensive sensor nodes. When the target is present, sensors observe an (unknown) deterministic signal with attenuation depending on the distance between the sensor and the (unknown) target positions, multiplicative fading (accounting for both line-of-sight and non-line-of-sight components), and additive Gaussian noise. To model energy-constrained operations usually encountered in an Internet of Things (IoT) scenario, local one-bit quantization of the raw measurement is performed at each sensor. The Fusion Center (FC) receives quantized sensor observations through error-prone binary symmetric channels and is in charge of performing a more-accurate global decision. Such model leads to a two-sided test with nuisance parameters (i.e. the target position \mathbf{x}_T) observable solely in the case of \mathcal{H}_1 hypothesis. After introducing the Generalized Likelihood Ratio Test (GLRT) for the problem, the appealing Davies’ framework is exploited to design a generalized form of the Rao test which obviates GLRT high complexity requirements. Equally important, a rationale for threshold-optimization (resorting to a heuristic principle) is proposed and confirmed via simulations. Finally, the aforementioned rules are compared in terms of detection rate in practical scenarios.

Keywords: Data fusion · Decentralized detection · GLRT · IoT · Rao test · Threshold optimization · Wireless sensor networks

1 Introduction

The Internet of Things (IoT) paradigm envisages billions of tiny devices with sensing, computation, and communicating capabilities to be used in numerous areas of everyday life [1]. These include Industry 4.0, smart cities and homes, precision-agriculture, healthcare, surveillance and security [2], just to name a few. In all these “verticals”, such devices are required to (a) measure the environment, (b) allow interaction with the physical world and (c) use Internet infrastructure

to provide services for data analytics, information transfer, and applications usage [3]. Wireless Sensor Networks (WSNs) constitute the sensing & actuation arm of the IoT and have attracted significant interest thanks to their flexibility and reduced costs [4, 5]. Decentralized detection is a key collective inference task for a WSN, which has been heavily investigated in the last decades [6].

Unfortunately, stringent bandwidth and energy constraints in WSNs hamper full-precision reporting by sensors. As a consequence, each node usually reports one bit to the Fusion Center (FC) regarding the inferred hypothesis. In such a case, the optimal sensor-individual decision procedure (from both Bayesian and Neyman-Pearson standpoints) corresponds to the local Likelihood-Ratio Test (LRT) being quantized into one-bit [7, 8]. Still, the design complexity of quantizer thresholds grows exponentially [9, 10] and, equally important, the evaluation of sensor LRT is precluded by ignorance of target parameters [10]. Hence, the bit reported is either the outcome of a naive quantization of the raw measurement [11, 12] or exemplifies the inferred binary-valued event (obtained via a sub-optimal detection statistic [13]). In both situations, FC gathers sensors bits and fuses them via a wisely-designed rule to improve (single-)sensor detection capability.

The optimum strategy to fuse the sensors' bits at the FC, under conditional independence assumption, is a weighted sum, with weights depending on unknown target parameters [6], except for some peculiar cases [14]. Then, simple fusion strategies, based on *simple* decisions' count rule or *simplifying* sensing model assumptions (at design stage), have been initially put forward to circumvent such unavailability [15–18]. Still, when the model is parametrically-specified (with some parameters unknown), the FC is in charge to tackle a composite test of hypotheses and the Generalized LRT (GLRT) is usually taken as the natural design solution [19]. Indeed, GLRT-based fusion of quantized data has been extensively studied in WSN literature [12, 20, 21], especially for decentralized detection of: (i) a cooperative target with unknown location, (ii) an uncooperative target modelled by observation coefficients assumed *known*, and (iii) an unknown source at unknown position (*uncooperative target*). Although case (iii) represents the most interesting and challenging (due to the least knowledge requirements), only a few works have recently dealt with it [11, 21–23]. In [21], a GLRT was derived for revealing a target with unknown position and emitted power and compared to the so-called counting rule, the optimum rule and a GLRT aware of target emitted power, showing a marginal loss of the proposed rule with respect to the latter GLRT form. Unfortunately, the designed GLRT requires a grid search on both the target location and emitted power (or signal) domains. Therefore, as a computationally simpler solution, generalized forms of score tests have been proposed for non-cooperative detection of either *deterministic* [11] or *stochastic* target emissions [23].

More recently, [24] and [25] have recently addressed the challenging multiplicative fading scenario in a decentralized estimation and detection problems. The latter setup indeed can be seen a generalization of both deterministic and stochastic emission models, and is able to model complicated propagation mech-

anisms, such as Rician models or imperfectly-estimated small-scale fading. However, for the multiplicative fading scenario, *only the simpler case (ii) has been addressed* (namely, detecting an unknown source with known observation coefficients).

To fill this gap, we focus on decentralized detection of a non-cooperative target with a spatially-dependent emission (signature), with emitted signal modelled as unknown and deterministic (as opposed to [22, 23]). More specifically, the received signal at each individual sensor experiences *multiplicative fading*, additive Gaussian noise, and a deterministic Amplitude Attenuation Function (AAF) depending on the sensor-target distance. Each sensor observes a local measurement on the absence/presence of the target and transmits a single bit version to a FC, over noisy reporting channels (modelled as Binary Symmetric Channels, BSCs, to emulate low-energy communications), having the task of a global (more accurate) decision output. The problem considered is a two-sided parameter test with nuisance parameters present only under the alternative hypothesis [26], which thus precludes the application of usual score-based tests, such as the Rao test [19]. In order to reduce the computational complexity required by the GLRT, the FC is designed to adopt a (simpler) sub-optimal fusion rule based on a generalization of the Rao test [11], and a *novel* quantizer threshold design is proposed herein, based on a *heuristic rationale* developed resorting to the performance of Position-Clairvoyant (PC) Rao in asymptotic form. The resulting design is *sensor-individual*, considers the channel status between each sensor and the FC, and depends upon neither the target strength nor its position, thus allowing *offline* computation. More important, we show zero-threshold choice is optimal according to the latter criterion. Finally, simulation results are provided to compare these rules in terms of performance and complexity in practical scenarios.

Paper Organization: Section 2 describes the system model; Sect. 3 develops GLR and G-Rao tests for the problem introduced; then, Sect. 4 focuses on optimization of the quantizer; numerical results are reported and discussed in Sect. 5; finally, concluding remarks (briefly highlighting further directions of research) are provided in Sect. 6.

List of Employed Math Notations: Bold letters in lower-case indicate vectors, with a_n representing the n th component of \mathbf{a} ; $\mathbb{E}\{\cdot\}$ and $(\cdot)^T$ are the expectation and transpose operators, respectively; the unit (Heaviside) step function is denoted with; $p(\cdot)$ and $P(\cdot)$ differentiate probability density functions (pdf) and probability mass functions (pmf), respectively; we denote a Gaussian pdf having mean μ and variance σ^2 with $\mathcal{N}(\mu, \sigma^2)$ is used to; $\mathcal{Q}(\cdot)$ (resp. $p_{\mathcal{N}}(\cdot)$) denotes the complement of the cumulative distribution function (resp. the pdf) of a normal random variable in its standard form, i.e. $\mathcal{N}(0, 1)$; finally, the symbol \sim (resp. $\stackrel{a}{\sim}$) corresponds to “distributed as” (resp. to “asymptotically distributed as”).

2 System Model

We focus on a binary test of hypotheses in which a set of nodes $k \in \mathcal{K} \triangleq \{1, \dots, K\}$ is displaced to monitor a given area to decide the absence (\mathcal{H}_0) or presence (\mathcal{H}_1) of a non-cooperative source with incompletely-specified spatial signature, and signal attenuation depending on the sensor-source distance and multiplicative fading, namely:

$$\begin{cases} \mathcal{H}_0 : & z_k = w_k \\ \mathcal{H}_1 : & z_k = g(\mathbf{x}_T, \mathbf{x}_k) h_k \theta + w_k \end{cases}, \quad (1)$$

In other terms, when the target is present (i.e. \mathcal{H}_1), we assume that its radiated signal θ , modelled as *unknown deterministic*, is isotropic and experiences (distance-dependent) path-loss, multiplicative fading and additive noise, before reaching individual sensors. In the test of hypotheses in Eq. (1), $z_k \in \mathbb{R}$ denotes the observation of k th sensor, whereas $w_k \sim \mathcal{N}(0, \sigma_{w,k}^2)$ and $h_k \sim \mathcal{N}(1, \sigma_{h,k}^2)$ the measurement noise and the multiplicative fading term, respectively. Additionally, $\mathbf{x}_T \in \mathbb{R}^d$ denotes the *unknown* position of the target, while $\mathbf{x}_k \in \mathbb{R}^d$ denotes the *known* k th sensor position (obtained via standard self-localization procedures). Both \mathbf{x}_T and \mathbf{x}_k *uniquely* determine the value of $g(\mathbf{x}_T, \mathbf{x}_k)$, generically denoting the AAF. We underline that we do not place any specific restriction regarding the AAF modelling the spatial signature of the target to be detected. In view of the spatial separation of the sensors, we hypothesize that the contributions due to noise and fading terms w_k s and h_k s are statistically independent.

Accordingly, the measured signal z_k is *conditionally* distributed as:

$$\begin{cases} z_k | \mathcal{H}_0 \sim \mathcal{N}(0, \sigma_{w,k}^2) \\ z_k | \mathcal{H}_1 \sim \mathcal{N}\left(g(\mathbf{x}_T, \mathbf{x}_k) \theta, g^2(\mathbf{x}_T, \mathbf{x}_k) \sigma_{h,k}^2 \theta^2 + \sigma_{w,k}^2\right) \end{cases}, \quad (2)$$

Then, to cope with stringent energy and bandwidth budgets in realistic IoT scenarios, the k th sensor quantizes z_k within one bit of information, i.e. $d_k \triangleq u(z_k - \tau_k)$, $k \in \mathcal{K}$, where τ_k represents the quantizer threshold (to be designed). For simplicity, we confine the focus of this paper to deterministic quantizers, while leaving the investigation of probabilistic quantizers to future studies. Additionally, with the aim of modeling a reporting phase with constrained energy, we assume that k th sensor bit d_k is transmitted over a BSC to the FC. Hence, due to non-ideal transmission, a possibly-erroneous $\hat{d}_k \neq d_k$ may be observed. In detail, we assume that bit flipping $\hat{d}_k = (1 - d_k)$ may occur with probability $P_{e,k}$, standing for the known bit-error probability of k th link. For notational convenience, we collect the WSN decisions received by the FC compactly as $\hat{\mathbf{d}} \triangleq [\hat{d}_1 \dots \hat{d}_K]^T$.

In view of the aforementioned assumptions, the bit probability under \mathcal{H}_1 is given by

$$\alpha_k(\theta, \mathbf{x}_T) \triangleq (1 - P_{e,k})\beta_k(\theta, \mathbf{x}_T) + P_{e,k}(1 - \beta_k(\theta, \mathbf{x}_T)), \quad (3)$$

where $\beta_k(\theta, \mathbf{x}_T) \triangleq \mathcal{Q}([\tau_k - g(\mathbf{x}_T, \mathbf{x}_k)\theta]/\sqrt{g^2(\mathbf{x}_T, \mathbf{x}_k)\sigma_{h,k}^2\theta^2 + \sigma_{w,k}^2})$. On the other hand, the bit probability when \mathcal{H}_0 holds is obtained as $\alpha_{k,0} \triangleq \alpha_k(\theta = 0, \mathbf{x}_T)$ (see Eq. (3)), simplifying into:

$$\alpha_{k,0} = (1 - P_{e,k})\beta_{k,0} + P_{e,k}(1 - \beta_{k,0}), \quad (4)$$

where $\beta_{k,0} \triangleq \beta_k(\theta = 0, \mathbf{x}_T) = \mathcal{Q}(\tau_k/\sqrt{\sigma_{w,k}^2})$.

We highlight that the unknown target position \mathbf{x}_T can be observed at the FC *only* when the *signal is present*, i.e. $\theta \neq \theta_0$ ($\theta_0 = 0$). Thus, we cast the problem as a two-sided parameter where parameters of nuisance (\mathbf{x}_T) are identifiable only under \mathcal{H}_1 [26]. In this paper, the pair $\{\mathcal{H}_0, \mathcal{H}_1\}$ corresponds to $\{\theta = \theta_0, \theta \neq \theta_0\}$. The objective of our study is tantamount to a simple test derivation (from a computational viewpoint) deciding for \mathcal{H}_0 (resp. \mathcal{H}_1) when the statistic $\Lambda(\hat{\mathbf{d}})$ is below (resp. above) the threshold γ_{fc} , and the design of the quantizer (i.e. an optimized τ_k , $k \in \mathcal{K}$) for each sensor.

Accordingly, we will evaluate FC system performance through the well-known detection ($P_D \triangleq \Pr\{\Lambda > \gamma_{fc}|\mathcal{H}_1\}$) and false alarm ($P_F \triangleq \Pr\{\Lambda > \gamma_{fc}|\mathcal{H}_0\}$) probabilities, respectively. In the previous definitions Λ denotes the generic decision statistic implemented at the FC.

3 Fusion Rules

The GLR represents a widespread technique for composite hypothesis testing [21], with its implicit form given by

$$\Lambda_{\text{GLR}}(\hat{\mathbf{d}}) \triangleq 2 \ln \left[\frac{P(\hat{\mathbf{d}}; \hat{\theta}_1, \hat{\mathbf{x}}_T)}{P(\hat{\mathbf{d}}; \theta_0)} \right], \quad (5)$$

where $P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T)$ represents the decision vector likelihood as a function of (θ, \mathbf{x}_T) . On the other hand, $(\hat{\theta}_1, \hat{\mathbf{x}}_T)$ are the Maximum Likelihood (ML) estimates under \mathcal{H}_1 , i.e.

$$(\hat{\theta}_1, \hat{\mathbf{x}}_T) \triangleq \arg \max_{(\theta, \mathbf{x}_T)} P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T), \quad (6)$$

with $\ln P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T)$ being the logarithm of the likelihood function vs. (θ, \mathbf{x}_T) , whose explicit form is [21, 22]

$$\ln P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T) = \sum_{k=1}^K \left\{ \hat{d}_k \ln [\alpha_k(\theta, \mathbf{x}_T)] + (1 - \hat{d}_k) \ln [1 - \alpha_k(\theta, \mathbf{x}_T)] \right\}, \quad (7)$$

and an analogous functional holds for $\ln P(\hat{\mathbf{d}}; \theta_0)$ by substituting $\alpha_k(\theta, \mathbf{x}_T) \rightarrow \alpha_{k,0}$. It is clear from Eq. (5) that Λ_{GLR} requires a maximization problem to be tackled. Sadly, an explicit expression for the pair $(\hat{\theta}_1, \hat{\mathbf{x}}_T)$ is not available.

This increases GLR complexity, since grid discretization of (θ, \mathbf{x}_T) is usually leveraged, see e.g. [21].

On the other hand, Davies’ work represents an alternative approach for capitalizing the two-sided nature of the considered hypothesis test [26], allowing to generalize Rao test to the more challenging scenario of nuisance parameters observed only under \mathcal{H}_1 . In fact, Rao test is based on ML estimates of nuisances under \mathcal{H}_0 [19], that sadly cannot be obtained, because they are not observable in our case. In detail, if \mathbf{x}_T were available, Rao fusion rule would represent an effective, yet simple, fusion statistic for the corresponding problem testing a two-sided hypothesis [19]. Unfortunately, since \mathbf{x}_T is not known in the present setup, we rather obtain a Rao statistics family by varying such parameter. Such technical difficulty is overcome by Davies through the use of the supremum of the family as the decision statistic, that is:

$$\Lambda_{\text{GRao}} \triangleq \max_{\mathbf{x}_T} \frac{\left(\frac{\partial \ln P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T)}{\partial \theta} \right)^2 \Big|_{\theta=\theta_0}}{I(\theta_0, \mathbf{x}_T)}, \tag{8}$$

where $I(\theta, \mathbf{x}_T) \triangleq \mathbb{E} \left\{ \left(\partial \ln [P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T)] / \partial \theta \right)^2 \right\}$ represents the Fisher Information (FI) assuming \mathbf{x}_T known. Henceforth, the above decision test will be referred to as Generalized Rao (G-Rao), to highlight the usage of Rao as the basic statistic within the umbrella proposed by Davies [22]. The closed form of Λ_{GRao} is drawn resorting to the explicit forms of the score function and the FI, as stated via the following lemmas, whose proof is omitted for brevity.

Lemma 1. *The score function $\partial \ln [P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T)] / \partial \theta$ for decentralized detection a non-cooperative target model with multiplicative fading obeys the following expression:*

$$\frac{\partial \ln [P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T)]}{\partial \theta} = \sum_{k=1}^K \left\{ \frac{\hat{d}_k - \alpha_k(\theta, \mathbf{x}_T)}{\alpha_k(\theta, \mathbf{x}_T)[1 - \alpha_k(\theta, \mathbf{x}_T)]} (1 - 2P_{e,k}) \times p_{\mathcal{N}} \left(\frac{\tau_k - g(\mathbf{x}_T, \mathbf{x}_k) \theta}{\sqrt{g^2(\mathbf{x}_T, \mathbf{x}_k) \sigma_{h,k}^2 \theta^2 + \sigma_{w,k}^2}} \right) \frac{g(\mathbf{x}_T, \mathbf{x}_k) \sigma_{w,k}^2 + g^2(\mathbf{x}_T, \mathbf{x}_k) \sigma_{h,k}^2 \theta \tau_k}{(\sigma_{w,k}^2 + g^2(\mathbf{x}_T, \mathbf{x}_k) \sigma_{h,k}^2 \theta^2)^{3/2}} \right\} \tag{9}$$

Proof. The proof can be obtained analogously as [11] by exploiting in the derivative calculation the separable form expressed by Eq. (7).

Lemma 2. *The FI $I(\theta, \mathbf{x}_T)$ for decentralized detection a non-cooperative target model with multiplicative fading has the following closed form:*

$$I(\theta, \mathbf{x}_T) = \sum_{k=1}^K \psi_k(\theta, \mathbf{x}_T) g^2(\mathbf{x}_T, \mathbf{x}_k), \tag{10}$$

where the following auxiliary definition has been employed

$$\psi_k(\theta, \mathbf{x}_T) \triangleq \frac{(1 - 2P_{e,k})^2}{\alpha_k(\theta, \mathbf{x}_T) [1 - \alpha_k(\theta, \mathbf{x}_T)]} \quad (11)$$

$$\times \frac{\left\{ \sigma_{w,k}^2 + g(\mathbf{x}_T, \mathbf{x}_k) \sigma_{h,k}^2 \theta \tau_k \right\}^2}{(\sigma_{w,k}^2 + g^2(\mathbf{x}_T, \mathbf{x}_k) \sigma_{h,k}^2 \theta^2)^3} p_{\mathcal{N}}^2 \left(\frac{\tau_k - g(\mathbf{x}_T, \mathbf{x}_k) \theta}{\sqrt{g^2(\mathbf{x}_T, \mathbf{x}_k) \sigma_{h,k}^2 \theta^2 + \sigma_{w,k}^2}} \right).$$

Proof. The proof can be obtained analogously as [11], exploiting conditional independence of decisions (which implies an additive FI form) and similar derivation results as $\frac{\partial \ln[P(\hat{\mathbf{d}}; \theta, \mathbf{x}_T)]}{\partial \theta}$.

Then, combining the results in (9) and (10), the G-Rao statistic is obtained in the final form as

$$\Lambda_{\text{GRao}}(\hat{\mathbf{d}}) \triangleq \max_{\mathbf{x}_T} \Lambda_{\text{Rao}}(\hat{\mathbf{d}}, \mathbf{x}_T), \quad (12)$$

where

$$\Lambda_{\text{Rao}}(\hat{\mathbf{d}}, \mathbf{x}_T) = \frac{\sum_{k=1}^K \hat{\nu}_k(\hat{d}_k) g(\mathbf{x}_T, \mathbf{x}_k)}{\sqrt{\sum_{k=1}^K \psi_{k,0} g^2(\mathbf{x}_T, \mathbf{x}_k)}}, \quad (13)$$

is the Rao statistic when \mathbf{x}_T is assumed known, and we have employed $\hat{\nu}_k(\hat{d}_k) \triangleq (\hat{d}_k - \alpha_{k,0}) \Xi_k$, $\psi_{k,0} \triangleq \alpha_{k,0} (1 - \alpha_{k,0}) \Xi_k^2$ and

$$\Xi_k \triangleq \frac{(1 - 2P_{e,k})}{\alpha_{k,0} (1 - \alpha_{k,0})} \frac{1}{\sigma_{w,k}} p_{\mathcal{N}} \left(\frac{\tau_k}{\sqrt{\sigma_{w,k}^2}} \right), \quad (14)$$

as compact auxiliary definitions. We motivate the attractiveness of G-Rao with a lower (resp. a simpler) complexity (resp. implementation), as we do not require $\hat{\theta}_1$, and only a grid search with respect to \mathbf{x}_T is imposed, that is

$$\Lambda_{\text{GRao}}(\hat{\mathbf{d}}) \approx \max_{i=1, \dots, N_{\mathbf{x}_T}} \Lambda_{\text{Rao}}(\hat{\mathbf{d}}, \mathbf{x}_T[i]). \quad (15)$$

Hence, the complexity of its implementation scales as $\mathcal{O}(K N_{\mathbf{x}_T})$, which implies a significant reduction of complexity with respect to the GLR (corresponding to $\mathcal{O}(K N_{\mathbf{x}_T} N_{\theta})$).

It is evident that Λ_{GRao} (the same applies to Λ_{GLR} , see Eqs. (5) and (13)) depends on τ_k 's, via the terms $\hat{\nu}_k(\hat{d}_k)$ and $\psi_{k,0}$, $k \in \mathcal{K}$. Hence the threshold set, gathered within $\boldsymbol{\tau} \triangleq [\tau_1 \cdots \tau_K]^T$, can be designed to optimize performance. Next section is devoted to accomplish this purpose.

4 Design of Quantizers

We point out that the rationale in [12, 27] cannot be applied to design (asymptotically) optimal deterministic quantizers, since no closed-form performance expressions exist for tests built upon Davies approach [26]. In view of this reason, we

use a modified rationale with respect to [12, 27] (that is resorting to a heuristic, yet intuitive, basis) and demonstrate its effectiveness in Sect. 5 through simulations, as done for similar uncooperative target detection problems in [11, 23]. In detail, it is well known that the (position \mathbf{x}_T) clairvoyant Rao statistic Λ_{Rao} is distributed (under an asymptotic, weak-signal, assumption¹) as [19]

$$\Lambda_{\text{Rao}}(\mathbf{x}_T, \boldsymbol{\tau}) \stackrel{a}{\sim} \begin{cases} \chi_1^2 & \text{under } \mathcal{H}_0 \\ \chi_1'^2(\lambda_Q(\mathbf{x}_T)) & \text{under } \mathcal{H}_1 \end{cases}, \quad (16)$$

where the non-centrality parameter $\lambda_Q(\mathbf{x}_T) \triangleq (\theta_1 - \theta_0)^2 \mathbb{I}(\theta_0, \mathbf{x}_T)$ (underlining dependence on \mathbf{x}_T) is given as:

$$\lambda_Q(\mathbf{x}_T) = \theta_1^2 \sum_{k=1}^K \psi_{k,0} g^2(\mathbf{x}_T, \mathbf{x}_k), \quad (17)$$

with θ_1 being the true value under \mathcal{H}_1 . Clearly the larger $\lambda_Q(\mathbf{x}_T)$, the better the \mathbf{x}_T -clairvoyant GLR and Rao tests will perform when the target to be detected is located at \mathbf{x}_T . Also, it is apparent that $\lambda_Q(\mathbf{x}_T)$ is a function of τ_k , $k \in \mathcal{K}$ (because of the $\psi_{k,0}$'s). For this reason, with a slight abuse of notation we will use $\lambda_Q(\mathbf{x}_T, \boldsymbol{\tau})$ and choose the threshold set $\boldsymbol{\tau}$ to maximize $\lambda_Q(\mathbf{x}_T, \boldsymbol{\tau})$, that is

$$\boldsymbol{\tau}^* \triangleq \arg \max_{\boldsymbol{\tau}} \lambda_Q(\mathbf{x}_T, \boldsymbol{\tau}) \quad (18)$$

In general, such optimization would lead to a $\boldsymbol{\tau}^*$ that will be dependent on \mathbf{x}_T (and thus not practical).

Still, for this particular problem, the optimization requires only the solution of K *decoupled* threshold designs (hence the optimization complexity presents a linear scale with the number of sensors K), being also independent of \mathbf{x}_T (cf. Eq. (17)), that is:

$$\tau_k^* = \arg \max_{\tau_k} \left\{ \psi_{k,0}(\tau_k) \propto \frac{p_{\mathcal{N}}^2(\tau_k / \sqrt{\sigma_{w,k}^2})}{\Delta_k + \mathcal{Q}(\tau_k / \sqrt{\sigma_{w,k}^2}) [1 - \mathcal{Q}(\tau_k / \sqrt{\sigma_{w,k}^2})]} \right\} \quad (19)$$

where $\Delta_k \triangleq [P_{e,k}(1 - P_{e,k})]/(1 - 2P_{e,k})^2$. It is known from the quantized estimation literature [28, 29] that for Gaussian pdf it holds $\tau_k^* \triangleq \arg \max_{\tau_k} \psi_{k,0}(\tau_k) = 0$. Also, it has been shown in [27] that $\tau_k^* = 0$ is still the optimal choice for any value of Δ_k , which corresponds to different possibilities of noisy ($P_{e,k} \neq 0$) reporting channels. Therefore, we employ $\tau_k^* = 0$, $k \in \mathcal{K}$, in Eq. (13), leading to the following further simplified expression for threshold-optimized G-Rao test (denoted with Λ_{GRao}^*):

$$\Lambda_{\text{GRao}}^* \triangleq \max_{\mathbf{x}_T} \frac{4 \left[\sum_{k=1}^K (1 - 2P_{e,k}) (1/\sigma_{w,k}) g(\mathbf{x}_T, \mathbf{x}_k) (\hat{b}_k - \frac{1}{2}) \right]^2}{\sum_{k=1}^K (1 - 2P_{e,k})^2 (1/\sigma_{w,k}^2) g^2(\mathbf{x}_T, \mathbf{x}_k)} \quad (20)$$

¹ That is $|\theta_1 - \theta_0| = c/\sqrt{K}$ for a certain value $c > 0$ [19].

which is considerably simpler than the GLRT, as it obviates solution of a joint optimization problem w.r.t. (\mathbf{x}_T, θ) . Furthermore, the corresponding optimized non-centrality parameter, denoted with $\lambda_Q^*(\mathbf{x}_T)$, is given by:

$$\lambda_Q^*(\mathbf{x}_T) \triangleq 4\theta_1^2 \sum_{k=1}^K [(1 - 2P_{e,k})^2 (1 / (2\pi\sigma_{w,k}^2)) g(\mathbf{x}_T, \mathbf{x}_k)^2]. \quad (21)$$

5 Simulation Results

Accordingly, in this section we delve into performance comparison of G-Rao and GLR tests. Additionally, we will provide an assessment of the threshold-optimization developed in Sect. 4. With this aim, a 2-D area $(\mathbf{x}_T \in \mathbb{R}^2)$ is considered, in which the presence of a non-cooperative target in the surveillance region $\mathcal{A} \triangleq [0, 1]^2$ (i.e. a square) is monitored by a WSN composed of $K = 49$ sensor nodes. For simplicity the sensors are arranged according to a regular square grid which covers the whole \mathcal{A} . With reference to the sensing model, we assume $w_k \sim \mathcal{N}(0, \sigma_w^2)$, $k \in \mathcal{K}$ (also w.l.o.g. we set $\sigma_w^2 = 1$). Also, the AAF chosen is $g(\mathbf{x}_T, \mathbf{x}_k) \triangleq 1 / \sqrt{1 + (\|\mathbf{x}_T - \mathbf{x}_k\| / \eta)^\alpha}$ (i.e. a power-law), where we have set $\eta = 0.2$ (viz. approximate target extent) and $\alpha = 4$ (viz. decay exponent). Finally, we define the target Signal-To-Noise Ratio (SNR), including multiplicative fading effects, as $\text{SNR} \triangleq 10 \log_{10}(\theta^2 (1 + \sigma_h^2) / \sigma_w^2)$ and the LoS/NLoS ratio as $\kappa \triangleq 10 \log_{10}(1 / \sigma_h^2)$. Initially, we assume ideal BSCs, i.e. $P_{e,k} = 0$, $k \in \mathcal{K}$.

According to Sect. 3, the implementation of Λ_{GLR} and Λ_{GRao} leverages grid search for θ and \mathbf{x}_T . Specifically, the search space of the target signal θ is assumed to be $S_\theta \triangleq [-\bar{\theta}, \bar{\theta}]$, where $\bar{\theta} > 0$ is such that the $\text{SNR} = 20$ dB. The vector collecting the points on the grid is then defined as $[-\mathbf{g}_\theta^T \ 0 \ \mathbf{g}_\theta^T]^T$, where \mathbf{g}_θ collects target strengths corresponding to the SNR dB values $-10 : 2.5 : 20$ (thus $N_\theta = 25$). Secondly, the search support of \mathbf{x}_T is naturally assumed to be coincident with the monitored area, i.e. $S_{\mathbf{x}_T} = \mathcal{A}$. Accordingly, the 2-D grid is the result of sampling \mathcal{A} uniformly with $N_{\mathbf{x}_T} = N_c^2$ points, where $N_c = 100$. The 2-D grid points are then obtained by regularly sampling \mathcal{A} with $N_{\mathbf{x}_T} = N_c^2$ points, where $N_c = 100$. In this setup, the evaluation of G-Rao requires $N_c^2 = 10^4$ grid points, as opposed to $N_c^2 N_\theta = 2.5 \times 10^5$ points for GLR, i.e. a complexity decrease of *more than twenty times*.

First, in Fig. 1 we show P_D (under $P_F = 0.01$) versus a common threshold choice for all the sensors $\tau_k = \tau$, $k \in \mathcal{K}$, for a target whose location is randomly drawn according to a uniform distribution within \mathcal{A} . We consider two LoS-NLoS conditions, namely $\kappa = 0$ dB (moderate LoS component) and $\kappa = 10$ dB (strong LoS component), whereas we fix the sensing $\text{SNR} = 10$ dB. It is apparent that in *both* LoS-NLoS conditions the choice $\tau_k^* = 0$ represents a nearly-optimal solution, since the optimal value of τ found numerically depends on the polarity of θ , which is *unknown*. This both applies to GLR and G-Rao as well. We then proceed with the choice $\tau_k^* = 0$ in following results.

Secondly, in Fig. 2 we provide a P_D comparison (for $P_F = 0.01$) of considered rules (for a target whose position is randomly generated within \mathcal{A} at each run,

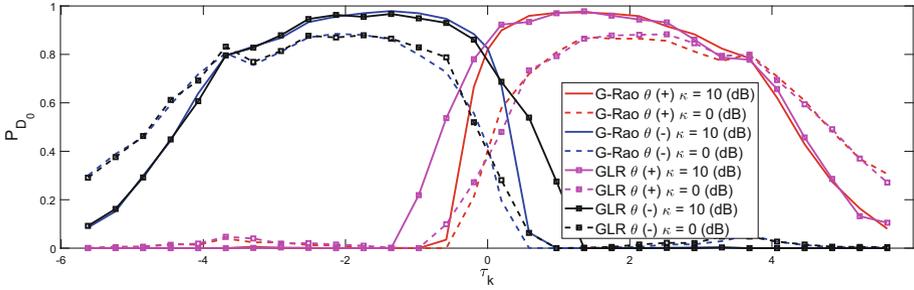


Fig. 1. P_D vs $\tau_k = \tau$, $P_F = 0.01$; WSN with $K = 49$ sensors, $P_{e,k} = 0$, SNR = 10 dB, $\kappa \in \{0, 10\}$ (amplitude signal θ with positive/negative polarity).

similarly as Fig. 1) versus SNR (dB), in order to assess their detection sensitivity as a function of the received signal strength of the non-cooperative target. We consider two relevant scenarios of LoS-NLoS ratio, namely $\kappa \in \{0, 10\}$ dB, and different quality of the BSC ($P_{e,k} = P_e \in \{0, 0.1\}$). From inspection of the figure, we conclude that both rules perform very similarly for a, different LoS-NLoS conditions, varying reporting channel status, and over the whole SNR range.

Thirdly, in Fig. 3, we report P_D (under $P_F = 0.01$) profile vs. target location \mathbf{x}_T (for SNR = 5 dB and $\kappa \in 0$ dB, $P_{e,k} = 0$), to draw a detailed overview of detection capabilities over the whole monitored area \mathcal{A} and underline possibly *blind spots*. Remarkably, G-Rao test performs only *negligibly worse* than the GLRT, and moderately worse in comparison to a test based on θ -clairvoyant GLR. Unfortunately, the latter assumes *the unrealistic* knowledge of θ and thus merely constitutes an *upper-limiting bound* on performance achievable. Finally, we notice that both GLR and G-Rao have a similar $P_D(\mathbf{x}_T)$ profile, and its “shape” highlights a lower detection rate at the edge of the monitored area. Such result can be ascribed to the *regularity* of the WSN arrangement in the monitored area \mathcal{A} .

6 Concluding Remarks and Further Directions

In this paper, a distributed scheme using a WSN for detection of a non-cooperative target was developed. More specifically, we considered a target emitting an unknown deterministic signal (θ) at unknown location (\mathbf{x}_T), and designed a generalized version of the Rao test (G-Rao) as an attractive (low-complexity) alternative to GLRT (the latter requiring a grid search on the whole space (θ, \mathbf{x}_T)) for a general model with (i) multiplicative fading, (ii) quantized measurements, (iii) non-ideal and non-identical BSCs. Since \mathbf{x}_T is a nuisance parameter present only under \mathcal{H}_1 (i.e. when $\theta \neq 0$), the G-Rao statistic arises from maximization (w.r.t. \mathbf{x}_T) of a family of Rao decision statistics, obtained by assuming \mathbf{x}_T known, based on the framework proposed by [26].

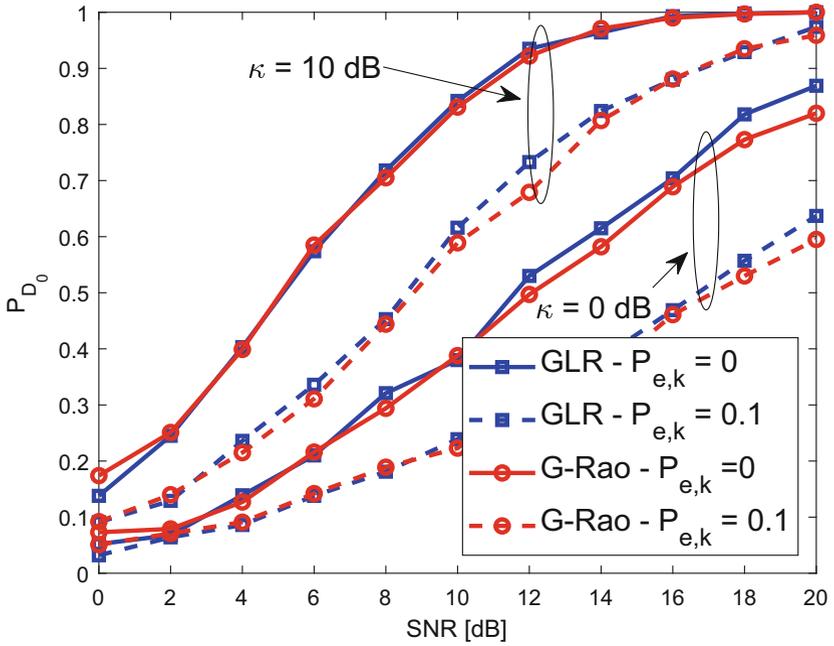


Fig. 2. P_{D_0} vs. sensing SNR (dB), when the FC false-alarm probability is set to $P_{F_0} = 0.01$. A WSN with $K = 49$ sensors is considered, with sensor thresholds set as $\tau_k^* = 0$, whose decisions are sent over BSCs with $P_{e,k} = P_e \in \{0, 0.1\}$ and with $\kappa \in \{0, 10\}$ (dB).

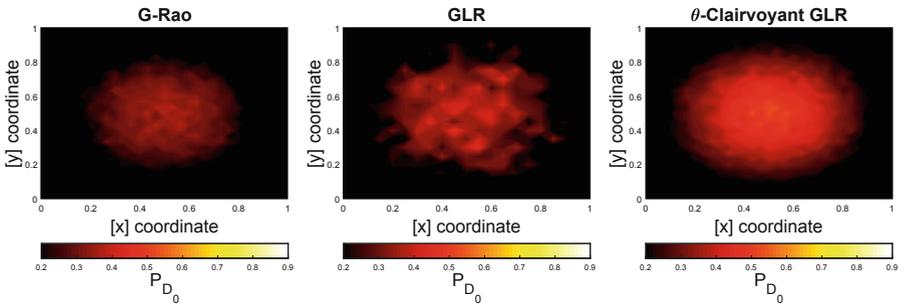


Fig. 3. G-Rao (left), GLR (center) and θ -clairvoyant GLR (right) P_D heatmaps vs. source position \mathbf{x}_T , when the FC false-alarm probability is set to $P_F = 0.01$. A WSN with $K = 49$ sensors, having sensing SNR = 5 dB and $\kappa = 0$ (dB) is considered. Corresponding decisions are sent over BSCs with $P_{e,k} = 0.1$. The sensor thresholds are set as $\tau_k^* = 0$.

Furthermore, we developed an effective criterion (originating from performance expressions having a semi-theoretical background) to design sensor thresholds of G-Rao in an optimized fashion, resulting in a *zero-threshold* choice. This result was leveraged to optimize the performance of G-Rao and GLR tests. Numerical results underlined the close performance of G-Rao test to the GLRT in the scenarios investigated, and a small (yet reasonable) loss of G-Rao compared to a test based on θ -clairvoyant GLR. Also, it was shown through simulations that the G-Rao test, achieves practically the same performance as the GLRT in the cases considered.

Our future work will consist of investigating design of fusion rules in more challenging target scenarios, such as multiple moving sources, multi-bit quantizers and burstiness (time-correlation) of reporting channels.

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